Determine os valores de ***A*** e ***B*** tais que

$$\frac{9}{x^2-11x+10}=\frac{A}{x-10}+\frac{B}{x-1}$$

e calcule

$$\frac{A-B}{2}$$

$$\frac{9}{x^2-11x+10}=\frac{A}{x-10}+\frac{B}{x-1}$$

$$\frac{9}{x^2-11x+10}=\frac{A(x-1)+B(x-10)}{(x-10)(x-1)}$$

$$\frac{9}{x^2-11x+10}=\frac{Ax-A+Bx-10B}{x^2-x-10x+10}$$

$$\frac{9}{\cancel{x^2-11x+10}}=\frac{x(A+B)+(-A-10B)}{\cancel{x^2-11x+10}}$$

$$\underbrace{9=x(A+B)+(-A-10B)}$$

$$A+B=0 \quad \to \quad A=-B$$

$$-A-10B=9 \quad \to \quad -(-B)-10B=9$$

$$\Rightarrow B-10B=9 \;\Rightarrow\; -9B=9 \;\Rightarrow\; B=\frac{9}{-9} \;\Rightarrow\; B=-1$$

Então, $A=-(-B)=-(-1)\Rightarrow A=1$

Logo, $\frac{A-B}{2}=\frac{1-(-1)}{2}=\frac{1+1}{2}=\frac{2}{2}=1$

Marque as divisões que são exatas, isto é, cujo resto é 0.

a) $\left(x^2-4x+4\right)\div x$

$$
\begin{array}{l|l}
\cancel{x^2}-4x+4 & x \\ \hline
\underline{\cancel{-x^2}\qquad\qquad} & x-4 \\
\quad\cancel{-4x}+4 & \\
\underline{\quad\cancel{+4x}\qquad} & \\
\qquad\qquad 4 &
\end{array}
$$

b) $\left(x^2-4x+4\right)\div\left(x-1\right)$

$$
\begin{array}{l|l}
\cancel{x^2}-4x+4 & x-1 \\ \hline
\underline{\cancel{-x^2}+x\qquad} & x-3 \\
\quad\cancel{-3x}+4 & \\
\underline{\quad\cancel{3x}-3} & \\
\qquad\qquad 1 &
\end{array}
$$

c) $\left(x^3-5x+7\right)\div\left(x-1\right)$

$$
\begin{array}{l|l}
\cancel{x^3}-5x+7 & x-1 \\ \hline
\underline{\cancel{-x^3}+x^2\qquad} & x^2+x-4 \\
\quad\cancel{x^2}-5x+7 & \\
\underline{\cancel{-x^2}+x\qquad} & \\
\qquad\cancel{-4x}+7 & \\
\underline{\qquad\cancel{4x}-4} & \\
\qquad\qquad\quad 3 &
\end{array}
$$

d) $\left(x^2-9\right)\div\left(x-3\right) \rightarrow$ divisão exata

$$
\begin{array}{r|l}
x^2-9 & x-3 \\ \hline
-x^2+3x & x+3 \\ \hline
\cancel{3x-9} & \\
\cancel{-3x+9} & \\ \hline
0 &
\end{array}
$$

$$\boxed{\frac{\left(x^2-9\right)}{\left(x-3\right)}=\frac{\cancel{\left(x-3\right)}\left(x+3\right)}{\cancel{\left(x-3\right)}}=x+3}$$

e) $\left(x^2-4x+4\right)\div\left(x-2\right)\rightarrow$ divisão exata

$$\boxed{\frac{\left(x^2-4x+4\right)}{\left(x-2\right)}=\frac{\left(x-2\right)^2}{\left(x-2\right)}=\frac{\cancel{\left(x-2\right)}\left(x-2\right)}{\cancel{\left(x-2\right)}}=x-2}$$

Reescreva a expressão $\dfrac{\sqrt[3]{7x}-4}{7x-4^3}, x \neq \dfrac{4^3}{7}$, de outra forma de modo que nessa outra expressão você possa calcular o seu valor quando $x=\dfrac{4^3}{7}$. Tal valor será dado por $\dfrac{u}{v}$. Calcule $u+v$.

- Chamando $y=\sqrt[3]{7x} \Rightarrow y^3=7x \Rightarrow x=\dfrac{y^3}{7}$
- Reescrevendo a expressão:

$$\frac{\sqrt[3]{7x}-4}{7x-4^3}=\frac{y-4}{\cancel{7}\left(\frac{y^3}{\cancel{7}}\right)-4^3}=\frac{y-4}{y^3-4^3}=\frac{\cancel{y-4}}{\cancel{(y-4)}\left(y^2+y\cdot 4+4^2\right)}=\frac{1}{y^2+4y+4^2}$$

$$\boxed{a^3-b^3=(a-b)\left(a^2+ab+b^2\right)}$$

- Voltando à variável x:

$$\frac{1}{y^2+4y+4^2}=\frac{1}{\left(\sqrt[3]{7x}\right)^2+4\sqrt[3]{7x}+4^2}$$

- Substituindo x por $\dfrac{4^3}{7}\left[x=\dfrac{4^3}{7}\right]$, vem:

$$\frac{1}{y^2+4y+4^2}=\frac{1}{\left(\sqrt[3]{7x}\right)^2+4\sqrt[3]{7x}+4^2}=\frac{1}{\left(\sqrt[3]{\cancel{7}\left(\frac{4^3}{\cancel{7}}\right)}\right)^2+4\sqrt[3]{\cancel{7}\left(\frac{4^3}{\cancel{7}}\right)}+4^2}=$$

$$=\frac{1}{\left(\sqrt[3]{4^3}\right)^2+4\sqrt[3]{4^3}+4^2}=\frac{1}{4^2+4\cdot 4+4^2}=\frac{1}{16+16+16}=\frac{1}{48}=\frac{u}{v}$$

- Logo, $u+v=1+48=49$

Considere um triângulo retângulo de catetos ***x*** e ***y***. Se $y=1$, ao subtrair a hipotenusa da expressão

$\dfrac{x^2}{y+\sqrt{x^2+y^2}}$, chegamos ao valor:

Dividindo-se o polinômio $x^6+19x+84$ por x^2-x+1

$$
\begin{array}{l|l}
\cancel{x^6}+19x+84 & x^2-x+1 \\ \hline
\underline{\cancel{-x^6}+x^5-x^4} & x^4+x^3-x-1
\end{array}
$$

$$\cancel{x^5}\ \cancel{-x^4}+19x+84$$

$$\underline{\cancel{-x^5}+\cancel{x^4}-x^3}$$

$$\cancel{-x^3}+19x+84$$

$$\underline{\cancel{x^3}-x^2+x}$$

$$\cancel{-x^2}+20x+84$$

$$\underline{\cancel{x^2}-x+1}$$

$$\underset{a}{19}x+\underset{b}{85} \Rightarrow 19+85=104$$

Se $x=1$ é raiz do polinômio: $2x^3-18x^2+46x-30$, a soma das outras duas raízes é:

$$2x^3-18x^2+46x-30 \quad \underline{\big|\quad x-1 \quad}$$

$a,b,c>0\ (a\neq b).$

$ab=\dfrac{c^2}{4}$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{\left(\sqrt{a}+\sqrt{b}\right)}{\left(\sqrt{a}-\sqrt{b}\right)}\times\frac{\left(\sqrt{a}+\sqrt{b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)}=\frac{a+\sqrt{ab}+\sqrt{ab}+b}{a+\cancel{\sqrt{ab}}-\cancel{\sqrt{ab}}-b}$$

$$=\frac{a+\sqrt{\frac{c^2}{4}}+\sqrt{\frac{c^2}{4}}+b}{a-b}=\frac{a+\frac{c}{2}+\frac{c}{2}+b}{a-b}=\frac{a+b+c}{a-b}$$